Reliability Assessment for Three-State Element Systems Using ARBITR Software

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Abstract: The article deals with reliability assessment methods for systems with three-state elements. It is shown that further development of conventional logic-and-probabilistic methods (LPM) eliminates the LPM deficiencies such as analysis of only two element states and assumption of their independency. Two models of element failure impact on the system reliability are shown which demonstrate the principles of reliability assessment using the ARBITR software which incorporates capabilities of algebra of disjoint event groups and is based on the general logic-and-probabilistic method (GLPM).

Keywords: Reliability, "fail-closed" mode, "fail-open" mode, group of disjoint events, functional integrity schemes, three-state mode.

1. INTRODUCTION

Reliability assessment for complex systems with elements which have more than two states is of significant interest. Despite many methods available for formalization of a mathematical model of such system functioning, the method based on algebra of disjoint event groups is a precise, rigorous method which is quite easy to deploy in the software. A distinguishing feature of reliability assessment for three-state element systems is application of so called schemes of functional integrity (SFI) which ensure system reliability modeling with consideration of fail-open and fail-closed modes. The method used for the system reliability analysis depends on the applied model of element failure impact on the system reliability.

2. MODEL 1

2.1 General Description

Dhillon and Singh [Dhillon et al. (1981)] suggest the following model of element failure impact on the system:

1. For series connection:
   - Open-failure of at least one element results in failure of the whole system;
   - Closed-failure of all elements results in failure of the whole system;
   - Closed-failure of a few elements with at least one working element does not result in the system failure.
2. For parallel connection
   - Closed-failure of at least one element results in failure of the whole system.

A general formula for system reliability was defined based on the analysis of failure-free operation of series and parallel systems. The function of system reliability consists of parallel minimum paths, and every single minimum path is a series connection of elements. Thus, this formula may be applied for any monotonous structure (series, parallel, bridge, etc.) with this model of element failure impact on the whole system.

For example, the expression for reliability of a bridge structure *Rb* with consideration of element open-failure and closed-failure modes, takes the form [(Dhillon et al., 1981, p.170)]:

$$R_b = 1 - \sum_{k=1}^{2} Q_{0K} = 1 - Q_{01} - Q_{02}, \quad (1)$$

where Q_{01}, Q_{02} — probabilities of the system open-failure (k=1) and closed-failure (k=2). Probabilities Q_{01} and Q_{02} are calculated by substituting elements open-failure (q_{oi}) and closed-failure (q_{si}) probabilities in the expression of the system reliability.

2.2 ARBITR Application For Task Decision

Reliability assessment of three-state element systems takes account of mutually exclusive elements failed-open and failed-closed events [(Dhillon et al., 1981, p.165)]. Thus we can rewrite the equation (1) as follows:

$$R_b = R_{CO} - Q_{CS} = 1 - (Q_{CO} - Q_{CS}), \quad (2)$$

Right part of the equation (2) corresponds to probability of the event which L-function is a conjunction of negation of two disjoint events [(Chercesov et al., 1991, p.51)]. Events

Preprint submitted to 2013 IFAC Conference on Manufacturing Modelling, Management, and Control.

are system failures due to "open" or "closed" states. This property of the group of disjoint events is shown in Appendix A.

To solve this task in ARBITR software, an algebra of groups of disjoint events offered by the general logic-and-probabilistic method (author — A.Mozhaev) is applied which may used for both individual events and events unified in equivalent schemes [Polenin et al. (2011)]. Schemes of functional integrity (SFI) are used as a graphic tool for formalized task statement and generation of systems of L-equations in ARBITR software. The task solution procedure in ARBITR comprises the following steps:

(1) Generation of the equivalent SFI (for element #1) to estimate the failed-open probability.
SFI structure is equal to the system structure. Probabilities of element failure-free operation only for open type failures are used as initial data, i.e. $p_i = 1 - q_i$.
(2) Generation of the equivalent SFI (for element #2) to estimate the failed-closed probability.
SFI structure is equal to the system structure. Probabilities of element closed failures (q_{si}) are used as initial data.
(3) Equivalent SFIs are combined as conjunction of negation of two disjoint events.

Fig. 1-3 show sample ARBITR screenshots for computing probability of the bridge structure failure for a system with three-state elements. The sequence of operations following the above said methodology is shown. Initial data for quantitative evaluation of probability of the system failure-free operation are similar to initial data shown in [Dhillon et al. (1981)], i.e. probabilities of open failure and closed failure are 0.2 and 0.1 correspondingly.

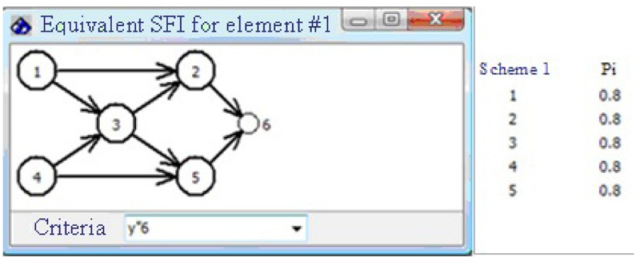

Fig. 1. Bridge structure. First step

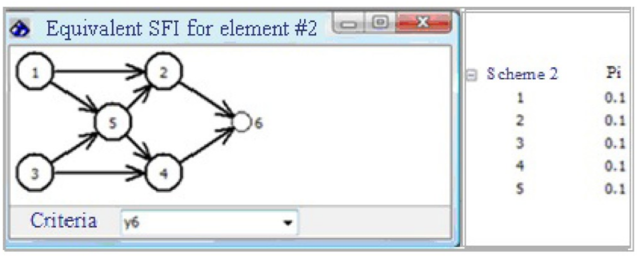

Fig. 2. Bridge structure. Second step

The results of probability evaluation for failure-free operation of the bridge structure Rb = 0.88984 meet the results shown in [Ryabinin (2007)].

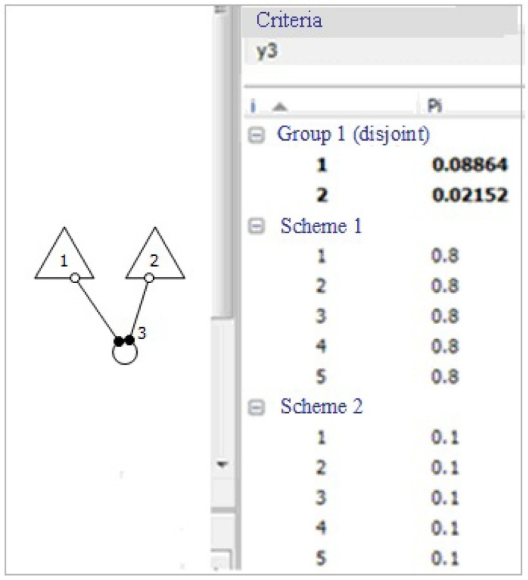

Fig. 3. Bridge structure. Thrid step

3. MODEL 2

3.1 General Description

The Professor Igor Ryabinin, while summarizing the analysis of LPM capabilities for three-state elements reliability analysis fairly noticed that "...there are particularly no actual systems with all elements having three states mode" [(Ryabinin, 2007, p.193)].

Therefore, let us consider an example of the power system comprising two generators (G1 and G2) and two power transmission lines (PTL3 and PTL4) [(Chercesov et al., 1991, p.47)]. They ensure redundancy of the main load power supply. Both generators may be either in operable (state X_1 and X_2) or in operable ($\bar{X}_1$ and $\bar{X}_2$) state. PTL3 and PTL4 are three-state elements: operable (X_3 and X_4) and inoperable. PTL failures may be either open failure (for PTL3 - X_5, for PTL4 - X_6) or closed failure (for PTL3 - X_7, for PTL4 - X_8). If at least one PTL is in fail-closed mode, the whole system fails.

3.2 ARBITR Application For Task Decision

SFI is generated to evaluate reliability of the power system in ARBITR software (Fig. 4).

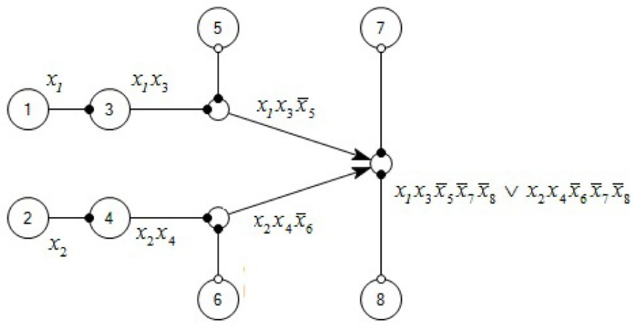

Fig. 4. SFI of the power system

The system of logical equations comprises logic variables x_i and $\bar{x}_i$, $i = \overline{1,8}$, which show (non)occurrence of the event i (transmission in i state), and it is written as a normal disjunctive form:

$$x_1 x_3 \bar{x}_4 \bar{x}_7 \bar{x}_8 \vee x_2 x_4 \bar{x}_6 \bar{x}_7 \bar{x}_8 \quad (3)$$

While decision making, it should be noted that all three states of PTL3 and PTL4 (states X_3, X_5, X_7 and X_4, X_6, X_8) represent complete groups of disjoint events and probability of each group is 1.

If we apply the Sylvester-Poincare formula to (3), we get the following probabilistic function to evaluate reliability of the power system:

$$R_c = P_1 P_3 Q_5 Q_7 Q_8 + P_2 P_4 Q_6 Q_7 Q_8 - \\ - P_1 P_2 P_3 P_4 Q_5 Q_6 Q_7 Q_8, \quad (4)$$

where R_c is reliability of the system;
P_i probability of i-th event occurrence;
$Q_i = 1 - P_i$.

Pursuant to the rule of computing probabilities for a product of two or more disjoint events - the rule of conjunction contraction [Appendix A] for groups of disjoint events (states X_3, X_5, X_7 and X_4, X_6, X_8), we get:

$$P_3 Q_5 Q_7 \Rightarrow P_3, P_4 Q_6 Q_8 \Rightarrow P_4. \quad (5)$$

Thus, (4) *with consideration of transformations* (5) will be as follows:

$$R_c = P_1 P_3 Q_8 + P_2 P_4 Q_7 - P_1 P_2 P_3 P_4. \quad (6)$$

For quantitative evaluation of system reliability, the following initial data are used [Chercesov et al. (1991)]: $P_1 = 0.6$; $P_2 = 0.4$; $P_3 = 0.5$; $P_4 = 0.7$; $P_7 = 0.2$; $P_8 = 0.2$; $P_5 = 1 - P_3 - P_7 = 0.3$; $P_6 = 1 - P_4 - P_8 = 0.1$. Therefore pursuant to (6), reliability of the power system with consideration of groups of disjoint events will be Rc =0.38. Reliability of the system without consideration of groups of disjoint events pursuant to (4) will be $R_c = 0.2618112$. Comparison of quantitative estimates of the power system reliability for this model of element failure impact on the overall system failure shows that consideration of mutually exclusive of fail-open and fail-closed modes results in reduction of the system reliability.

4. CONCLUSION

Conventional logic-and-probabilistic methods are used as a basis for development of methods of reliability evaluation for systems with three-state elements. The apparatus of groups of disjoint events deployed in the ARBITR software, is used as a theoretical basis for this methodology [Chercesov et al. (1991), Nozik (2005), Polenin et al. (2011)]. It is shown that the general logic-and-probabilistic approach may be used for the tasks when the elements have more than two states, and changes of element states represent stochastically dependent events.

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Appendix A. TRANSFORMATION OF LOGIC AND PROBABILISTIC FUNCTIONS

The Appendix shows substantiation and several evidences of transformation of logic functions (L-functions) and probabilistic functions (P-functions). To find substantiation and evidence of all 24 transformations of L-functions and P-functions, please visit http://www.szma.com/pub.shtml.

A.1 Conjunctions on disjoint events

Transformations for this group of conjunctions are shown in lines 1.1, 2.2, and 3.4 of consolidated tables B.1 - B.3. Evidence of consistency of a transformation is given for mutually exclusive *direct* events a and b. Table A.1 shows a truth table and an L-function transformation for the said group of disjoint events.

Table A.1. Truth table for conjunction $a \wedge b$

a	b	ab	Tr	TLF
0	0	0		0
0	1	0		0
1	0	0		0
1	1	1	$1 \Rightarrow 0$	0

Column ab of the table A.1 shows results of conjunction without consideration of mutually exclusive of events a and b. If we replace "1" onto "0" in the last line of this column which corresponds to disjoint direct events (*Tr* column which stands for transmission), then in the last column *"TLF"* (transformed L-function), the logic function with consideration of the group of disjoint events will be equal to logic "0". This transformation is explained by the definition of mutual exclusivity of a and b events. Therefore, logic transformation for this conjunction may be expressed as follows:

$$a \wedge b \Rightarrow 0, \quad (A.1)$$

and the probabilistic transformation will be as follows:

$$P(ab) = 0. \quad (A.2)$$

A.2 Conjunction contraction

Transformations for this conjunction group are shown in lines 1.2, 1.3, 2.1, 2.4, 3.2, and 3.3 of tables B.1 - B.3.

Explanation of accuracy of this transformation is shown for mutually exclusive direct events a and b with regard to conjunctions of events $\bar{a}$ and b.

Events $\bar{a}$ and b *are not disjoint events*, but to evaluate the probability $P(\bar{a}b)$ of their product, it is necessary to take in account mutually exclusive of their direct outcomes.

Venn diagram for this case is shown in fig. A.1. As it is shown in fig. A.1, a set (event) b is an intersection of sets (events) $\bar{a} \cap b$.

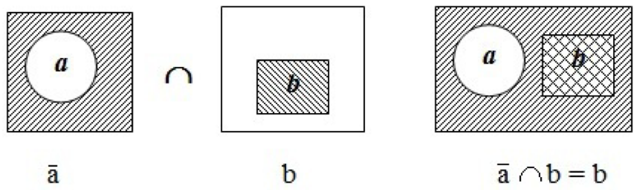

Fig. A.1. Venn diagram. Intersection of events $\bar{a} \cap b = b$ (disjoint direct events)

Thus, taking in consideration the mutually exclusive of direct events, L-function of the conjunction is transformed as follows:

$$\bar{a} \wedge b \Rightarrow b, \quad (A.3)$$

which corresponds to P-function transformation:

$$P(\bar{a}b) = P(b). \quad (A.4)$$

Table A.2 shows a truth table and a transformation of L-function for the considered group of disjoint events.

Table A.2. Truth table for conjunction $\bar{a} \wedge b$

a	b	$\bar{a}$	$\bar{a}b$	Tr	TLF
0	0	1	0		0
0	1	1	1		1
1	0	0	0		0
1	1	0	0	$0 \Rightarrow 1$	1

Column *"Tr"* of the Table A.2, as well as in Table A.1 shows transformation of the events product mentioned in the last line, which meets the condition of mutually exclusive *direct* events. With consideration of this mutually exclusive, the L-function shown in column "TLF" coincides with L-function of the event b.

A.3 Conjunction transformation using the law of dualization (De Morgan's theorem)

Transformations for this conjunction group are shown in lines 1.4, 2.3, and 3.1 of tables B.1 - B.3. Explanation of accuracy of this transformation is shown for mutually exclusive events $\bar{a}$ and b with regard to conjunctions of events a and $\bar{b}$.

With consideration of mutually exclusive events $\bar{a}$ and b, L-function for the conjunction $a \wedge \bar{b}$ may be written as follows:

$$a \wedge \bar{b} \Rightarrow \overline{\bar{a} \vee b}, \quad (A.5)$$

and the P-function will be as follows:

$$P(a \wedge \bar{b}) = 1 - [P(a) + P(\bar{b})]. \quad (A.6)$$

Table A.3. Truth table for disjunction $a \vee b$

a	b	$a \vee b$	Tr	TLF
0	0	0		0
0	1	1	$1 \Rightarrow 0$	0
1	0	1		1
1	1	1		1

A.4 Disjunction contraction

Transformations for this disjunction group are shown in lines 1.6, 1.7, 2.5, 2.8, 3.6, and 3.7. of tables B.1 - B.3.

Explanation of accuracy of this transformation is shown for mutually exclusive events $\bar{a}$ and b with regard to disjunctions of events a and b. Venn diagram for this case is shown in fig. A.2.

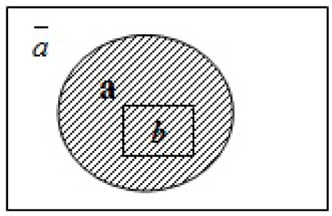

Fig. A.2. Venn diagram. Union of events $a \vee b$

Diagrams in fig. A.2 show that with consideration of mutually exclusive events $\bar{a}$ and b, L-function for this disjunction may be transformed as follows:

$$a \vee b \Rightarrow a, \quad (A.7)$$

which corresponds to transformation of P-function as follows:

$$P(a \vee b) = P(a). \quad (A.8)$$

Table A.3 shows a truth table for the considered group of disjoint events.

Table A.3 shows transformation of a product of events specified in the second line, which meets the condition of mutually exclusive events $\bar{a}$ and b. As it is shown in "TLF" column, L-function of the disjunction $a \vee b$ with consideration of mutually exclusive events $\bar{a}$ and b coincides with LF-function of the event a.

Transformations of groups 2 and 4 are called the disjoint absorption law.

A.5 Transformation of events disjunction into the universe

Transformations for this disjunction group are shown in lines 1.8, 2.7, and 3.5 of tables B.1 - B.3. Explanation of accuracy of this transformation is shown for mutually exclusive inverse events $\bar{a}$ and $\bar{b}$ with regard to disjunction of events a and b. Table A.4 shows a truth table for the considered group of disjoint events. Table A.4 shows that logic transformation for this disjunction may be written as follows:

$$a \vee b \Rightarrow 1, \quad (A.9)$$

and the probabilistic transformation will be as follows:

$$P(a \vee b) = 1. \quad (A.10)$$

Preprint submitted to 2013 IFAC Conference on Manufacturing Modelling, Management, and Control.

Table A.4. Truth table for disjunction $a \vee b$

a	b	$a \vee b$	Tr	TLF
0	0	0	$0 \Rightarrow 1$	1
0	1	1		1
1	0	1		1
1	1	1		1

Transformation of this group of L-functions is called the disjunctions truth law.

A.6 Disjunctions of disjoint events

Transformations for this disjunction group are shown in lines 1.5, 2.6 and 3.8 of table B.1 - B.3. L-function is not transformed, and P-function *by definition* is equal to probability of a sum of disjoint events.

Appendix B. CONSOLIDATED TABLES OF LOGIC AND PROBABILISTIC FUNCTION TRANSFORMATIONS

The consolidated tables B.1 - B.3 show transformations of L-functions and P-functions which correspond to various combinations of disjoint events — both direct and inverse. Transformed L-functions (TLF) and transformed P-functions (TPF) account for the type of mutually exclusive events shown in the title of each table.

Table B.1. Mutually exclusive direct events a and b

#	*LF*	*TLF*	*PF*	*TPF*
Conjunctions				
1.1	ab	0	$P(ab)$	0
1.2	$\bar{a}b$	b	$P(\bar{a}b)$	$P(b)$
1.3	$a\bar{b}$	a	$P(a\bar{b})$	$P(a)$
1.4	$\bar{a}\bar{b}$	$\overline{a \vee b}$	$P(\bar{a}\bar{b})$	$1-[P(a)+P(b)]$
Disjunctions				
1.5	$a \vee b$	$a \vee b$	$P(a \vee b)$	$P(a)+P(b)$
1.6	$\bar{a} \vee b$	$\bar{a}$	$P(\bar{a} \vee b)$	$P(\bar{a})$
1.7	$a \vee \bar{b}$	$\bar{b}$	$P(a \vee \bar{b})$	$P(\bar{b})$
1.8	$\bar{a} \vee \bar{b}$	1	$P(\bar{a} \vee \bar{b})$	1

Table B.2. Mutually exclusive events $\bar{a}$ and b

#	*LF*	*TLF*	*PF*	*TPF*
Conjunctions				
2.1	ab	b	$P(ab)$	$P(b)$
2.2	$\bar{a}b$	0	$P(\bar{a}b)$	0
2.3	$a\bar{b}$	$\overline{\bar{a} \vee b}$	$P(a\bar{b})$	$1-[P(\bar{a})+P(b)]$
2.4	$\bar{a}\bar{b}$	$\bar{a}$	$P(\bar{a}\bar{b})$	$P(\bar{a})$
Disjunctions				
2.5	$a \vee b$	a	$P(a \vee b)$	$P(a)$
2.6	$\bar{a} \vee b$	$\bar{a} \vee b$	$P(\bar{a} \vee b)$	$P(\bar{a})+P(b)$
2.7	$a \vee \bar{b}$	1	$P(a \vee \bar{b})$	1
2.8	$\bar{a} \vee \bar{b}$	$\bar{b}$	$P(\bar{a} \vee \bar{b})$	$P(\bar{b})$

Table B.3. Mutually exclusive direct events a and b

#	*LF*	*TLF*	*PF*	*TPF*
Conjunctions				
3.1	ab	$\overline{\bar{a} \vee \bar{b}}$	$P(ab)$	$1-[P(\bar{a})+P(\bar{b})]$
3.2	$\bar{a}b$	$\bar{a}$	$P(\bar{a}b)$	$P(\bar{a})$
3.3	$a\bar{b}$	$\bar{b}$	$P(a\bar{b})$	$P(\bar{b})$
3.4	$\bar{a}\bar{b}$	0	$P(\bar{a}\bar{b})$	0
Disjunctions				
3.5	$a \vee b$	1	$P(a \vee b)$	1
3.6	$\bar{a} \vee b$	b	$P(\bar{a} \vee b)$	$P(b)$
3.7	$a \vee \bar{b}$	a	$P(a \vee \bar{b})$	$P(a)$
3.8	$\bar{a} \vee \bar{b}$	$\bar{a} \vee \bar{b}$	$P(\bar{a} \vee \bar{b})$	$P(\bar{a})+P(\bar{b})$