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LOGICAL PROBABILISTIC ANALYSIS AND ITS HISTORY

CONTENTS

Introduction

Finding analogies between the mathematical logic and the probability theory is both of theoretical and practical importance. George Boole in "An Investigation of the Laws of Thought on Which are Founded the Mathematical Theories of Logic and Probabilities" published in London in 1854 expressed a far-sighted idea on applicability of the mathematical logic to the probability theory.

It is therefore astonishing, that during the last one and a half centuries not a single mathematician expressed his/her opinion on this matter, which gave reason to critics who said there is no mention on any relation between this science and the algebra of logic in any large publications on the probability theory, in particular, in publications of such academicians as A.A. Markov, A.N. Kolmogorov, etc.

What is a phenomenon of the logical probabilistic analysis and why it is neglected by mathematicians?

1. Founder of the Logical Probabilistic Analysis

George Boole (2 November 1815 – 8 December 1864), an English mathematician and logician, was at the very beginning of this history. His book "An Investigation of the Laws of Thought on Which are Founded the Mathematical Theories of Logic and Probabilities" [1], published in London in 1854 contains his far-sighted idea on applicability of the mathematical logic to the probability theory.

In his book [2], unpublished during his lifetime he mentioned applicability of probabilistic assessment for logic propositions, i.e. possible numerical expression of propositions.

Boole laid the grounds for the mathematical logic. Boolean algebra was named after George Boole.

However, it is Platon Sergeevich Poretskii (3 October 1846 – 9 August 1907), a Russian mathematician and logician who should be named the founder of the Logical Probabilistic Analysis (LPA). On 25 October 1886 in his report ("Solution of General Tasks in Probability Theory Through the Mathematical Logic" [3]) Poretskii gave scientific form to Boole's idea on applicability of mathematical logic to the probability theory.

In paragraph 1 of his work [3] he raises a philosophical question: is it possible to apply the theory of qualitative symbols (logical classes) to the theory of quantitative (probabilistic) symbols?

"Of course, it is possible", he resumes.

Page 3 [3] shows the first definition of the LPA: "…This opens a common way for definition of probabilities, i.e. finding a logical connection between the event, which probability is defined, and other events, which probabilities are given, and then make ***a transition*** from a logical equation of the events to an algebraic equation of their probabilities".

Highlighting the "*transition*" notion makes this definition very special, anticipating the description of the major result of his work. To apply **the law of inconsistency** one should know how to transform each logical polynomial

$$A \vee B \vee C \vee D \vee \ldots \qquad (1)$$

to a disjoint (or in modern Russian terminology – orthogonal) expression, i.e. to:

$$A \vee \bar{A}B \vee \bar{A}\bar{B}C \vee \bar{A}\bar{B}\bar{C}D \vee \ldots, \qquad (2)$$

where $\bar{A}$ is a negation of A, $\bar{B}$ is a negation of B, etc.

In this article, I used the modern rules to denote a logical sum $\vee$ and negation $\bar{A}$ ("+" and *Ao* correspondingly in the Poretskii's work).

Both polynomials (1) and (2) are logically equivalent, however the rule above may be applied to the first polynomial only.

Probability of the logical sum

$$P(A \vee B \vee C \vee D),$$

reduced to a disjoint expression is divided into an arithmetic sum of probabilities

$$P(A) + P(\overline{A}B) + P(\overline{A}\overline{B}C) + P(\overline{A}\overline{B}\overline{C}D). \tag{3}$$

From the theory of probabilities it is known that if two or more events are independent, then probability of their product is a product of their probabilities. That is, if *a, b, c …* are elementary events without any logical relations, then

$$P(abc\ldots) = P(a)P(b)P(c)\ldots$$

and

$$P(A) + P(\overline{A})P(B) + P(\overline{A})P(\overline{B})P(C) + \ldots \tag{4}$$

2. Poretskii's Orthogonalization Algorithm (1886)

Page 7 shows an original orthogonalization algorithm for a disjunction (algorithm of orthogonalization)

$$ab \vee cd. \tag{5}$$

1) **External orthogonalization** (outer loop procedure) is made

$$ab \vee \overline{ab}cd. \tag{6}$$

2) Negation $\overline{ab}$ is transformed in disjunction of two negations based on the de Morgan Law

$$\overline{ab} = \overline{a} \vee \overline{b}. \tag{7}$$

3) **Internal orthogonalization** is made.

$$\overline{a} \vee \overline{b} = \overline{a} \vee a\overline{b}. \tag{8}$$

4) All three operations are unified

$$\begin{aligned} ab \vee cd &= ab \vee \overline{ab}cd = ab \vee (\overline{a} \vee \overline{b})\, cd = \\ &= ab \vee (\overline{a} \vee a\overline{b})\, cd = ab \vee \overline{a}cd \vee a\overline{b}cd. \end{aligned} \tag{9}$$

The last expression is an Orthogonal Disjunctive Normal Form (ODNF) which helps to calculate the probability value

$$P(ab \vee cd) = P(ab) + P(\overline{a}cd) + P(a\overline{b}cd). \tag{10}$$

Therefore, Platon Poretskii in 1886 opened a rigorous mathematical method for calculation of complex event probability through probability of elementary events.
Evolution of ideas of mathematical logic was not a rising pattern, having its highs and lows, booming periods being often replaced by regression and partial decay.

It may be of service to remind the critics of a Moscow logician and mathematician, B.M. Koyalovich [4, p. 417], who actively criticized practical inefficiency of the algebra of logic, which, in his opinion, was not able to produce any fruitful extra logical applications. The professor I.V. Sleshinskiy, who translated into Russian the book of Louis Couturat “The Algebra of Logic”, in his reply to Koyalovich, mentioned the Poretskii’s work “Solution of General Tasks in Probability Theory Through the Mathematical Logic” where Poretskii had slightly rationalized and further developed the Boole’s ideas.

Furthermore, Koyalovich referred to the fact that none of the works on the probability theory of that time (including the monograph [5] by the Russian Academician A.A. Markov) contains any mention on communication of this scientific discipline with the algebra of logic.

However, in 1910 the physicist Paul Ehrenfest was first to propose to apply mathematical logic in technics: "The symbolical formulation will give the chance to "calculate" consequences from such difficult assumptions which in their verbal form are almost or absolutely impossible to understand". He showed his idea on the example of wire circuits of automated telephone system.

3. S.N. Bernstein’s Axiomatic of the Propositional Logic for Axiomatization of the Probability Theory (1917)

Only S.N. Bernstein was not concerned about formal difference between qualitative and quantitative symbols, and 30 years later he developed the first axiomatic of the propositional logic for axiomatization of the probability theory [6].

Valery Glivenko in his work [10, p. 274] wrote: “S.N. Bernstein proposed to consider the probabilities as probabilities of the truth of propositions. This makes it unnecessary to define special axiomatic for the events notion, and we may use the ready axiomatic of propositions”.

I think that Sergey Bernstein knew works of Platon Poretskii [3], who in 1870 graduated from the Faculty of Physics and Mathematics of the Kharkov University, where Bernstein was teaching from 1907 to 1933.

No practical demand on these ideas (in the first half of XX century) resulted in neglect of prominent mathematical ideas by Poretskii and Bernstein. This trend was supported by difficulty in finding their publications. By referencing the publication [3] which name was mentioned in [7,8,9], as origins of the logical probabilistic method, I still was far from identifying Platon Poretskii as a founder of the logical probabilistic analysis.

4. Mathematical Logic Prosperity and Difficulties in the USSR in the Mid XX Century

In 30s and further decades, in the USSR the logic required protection both in philosophical and mathematical communities.

In 1938, in a joint encyclopedic article devoted to the mathematical logic Sofya A. Yanovskaya with Valery I. Glivenko declared the mathematical logic to be just a science of reasoning in mathematics. This "interpretation" of the logic theory prevented her from attacks of followers of the **“**dialectical logic**”**. In 1943, she arranged a scientific workshop on the mathematical logic at the Faculty of Mechanics and Mathematics of the Moscow State University and she directed this workshop together with I.I. Zhegalkin and P.S. Novikov. Her works devoted to the mathematical logic resulted in opening of the Department of Mathematical Logic on March 3, 1959 at the faculty. There she acted as a main organizer and worked as a professor of that Department till the very end of her life (October 24, 1966).

Andrey A. Markov (September 22, 1903 – October 11, 1979) was the first Head of this Department. Andrey N. Kolmogorov (April 25, 1903 – October 20, 1987) headed this Department from January 1980 till October 1987.

In his big article written in 1947 on Mathematics in Its Historical Development [11], Kolmogorov defines the roles of the mathematical logic in a single paragraph.

“These studies result in a big independent section of mathematical science, i.e. Mathematical Logic. The grounds of mathematical logics were laid in XIX century by George Boole, Platon Poretskii, E.Schreuder, G. Frege, G. Peano, etc.” Nothing else is said about Platon Poretskii or his contribution to the logical probabilistic analysis.

In 1938, V. Shestakov and K. Shennon gave a complete proof of the possibility to use the propositional calculus to describe relay switching circuits. Based on these works, Mikhail A.Gavrilov developed harmonious theory of analysis and synthesis of relay switching circuits, where the functions recorded in a disjunctive normal form (DNF) shall be transformed into the orthogonal disjunctive normal form (ODNF), named a canonical form by Gavrilov [12, page 213].

In 1956 advanced Boole’s ideas on expansion of the formalism of the algebra of logic to the probabilities facilitated publication by N. Rouche [13], a French mathematician who defines how to change the perfect disjunctive normal form (PDNF) in order to replace the logical variables with their probabilities, and thus receive a probability of implementation of a complex proposition.

5. Secondary Opening of the Orthogonalization Algorithm by Yu.V. Merekin (1963)

The algorithm of orthogonalization was opened for the second time in 1963 at the Computer Department of the Institute of Mathematics (Novosibirsk) by Yuriy V. Merekin, a specialist in computing devices. At that time the probability of calculation of the address to unit of Boolean function (N.Rouche) was already easy to find. For the solution of applied problems, application of the PDNF was considered to be a trivial solution due to a large number of disjunctive members. It became necessary to create a short orthogonal form which was received in 1963 [14]. In reply to my request regarding Platon Poretskii sent to Yuriy Merekin in April 2011, he wrote on May 03. 2011: "I have not used the book of Platon Poretskii published in 1887".

For the third time the orthogonolization was independently opened in 1973 by Italians Luidge Fratta and Ugo Montanari [15] using Karnaugh maps and a concept of Disjoint Products

$$DP = Pi \ \& \ Pj = 0.$$

Logical probabilistic analysis was developed by engineers [4, 7, 8, 9, 12], while "pure" mathematicians have not shown any interest to this problem yet. Therefore, the critical comments with regards to the logical and probabilistic analysis similar to that of Koyalovich in early XX century pretending that no publications on the probability theory contain information on relationship between probability and the algebra of logic, continue to appear even now.

I may only regret that at the end of XX century, there is neither scientific estimate of the LPA nor any mention of Poretskii's name in the book "Mathematical Logic" [16] by A.N. Kolmogorov, A.G. Dragalin.

As I. Sleshinskiy rightfully wrote in the obituary notice on August 10, 1907, the name of Platon Poretskii was better known abroad than in his country.

6. LPA Critics in Modern Mathematics (1998-2000)

In this context, the criticism of LPA by modern mathematicians [17, 18] in the spirit of denial of possibility of application of the doctrine about qualitative characters to the doctrine about quantitative characters is not surprising.

The professor Ya.Ya. Golota considers that "The algebra of propositional logic takes as a point of departure full definition of the objects under study. The probability theory assumes uncertainty of the events. Thus, one theory unifies the contrary assumptions – full definition and uncertainty. Doesn't it mean an evident contradiction which lies in the very grounds of the logic-probabilistic theory?" Besides, he says [17]: "And the concept of "probability" itself which embodied many years of its usage, does not let us think that traditional definition of probability permits to understand it as an assessment of the truth of expressions".

Another Doctor of Engineering, V.N. Sokolyuk in his work [18, p. 103], reasoning about the logics compliance to the general paradigm, writes: "If we follow the probability theory, then we must admit that it is an absurd to speak about "the truth of events" and "probability of propositions", since the truth refers to propositions, and not the events, and probability refers to events, and not the propositions. Each proposition is phenomenal. It does not make sense to discuss their large-scale involvement in theoretical and probability aspects, though many "scientists" even write books and create theories which lead to nowhere [18]".

7. The Need in LPA Application for Solving Reliability, Survivability and Safety Problems

To get acquainted with the theories which lead to nowhere in Russian practice, I would recommend the article [20], which refers to 42 sources and describes the logical and probabilistic analysis as a tool for research of reliability, survivability, and safety of structurally complex systems of a wide variety of types.

Even more examples of LPA application in international publications are offered in [21,22,23]. They are defined there are:

- A Boolean Algebra Method [BAM];
- Disjoint Boolean Products [DBP];
- Boolean Function Manipulations [BFM];
- Logical and Probability Analysis [LPA];
- Sum of Disjoint Products [SDP].

In the 2nd half of XX century LPA became more needed for purposes of estimating the reliability and safety measures of structurally-complex systems of any type. Such increased interest in LPA was due to the computer development, i.e. computerization of LPA methods.

Boole's idea about applicability of the mathematical logic to the probability theory resulted to both LPA and probabilistic logic (PL) development.

The logic of probabilities (LP) deals with calculation of a probability of the truth of propositions, which take only two values – true (1) or false (0).

The probabilistic logic deals with assessment of the truth of propositions (hypotheses) which take a variety of values within the given range ($0 \leq x \leq 1$).

In other words, in the former case we have a binary logic, while in the latter one – a multi-valued logic. Therefore, the logic of probabilities is naturally simpler than the probabilistic logic.

All formulas of the probability theory for complex events $y=f(x_1,\dots,x_n)$, which are functions of independent events $x_1,\dots,x_n$, become correct logic formulas when the events are replaced with appropriate propositions.

This remarkable phenomenon, as described by a Doctor of Physics and Mathematics, Professor Nikolay V. Khovanov takes place since the propositions have only two values of truth: (false) 0, (truth) 1.

If we take empty $\varnothing$ and exhaustive Ω events with the probabilities $P\{\varnothing\}=0$ and $P[\Omega]=1$, they are independent from other events, from each other and from themselves.

Back on the applicability of the theory of qualitative symbols to the theory of quantitative symbols, we should pay attention to the notion *"proposition"*.

What kinds of propositions, expressions or judgments are described in the logic-probabilistic analysis? Of course, not all of them, but only those which may answer to Yes/No questions, i.e. contradictory oppositions, and not contrary ones. This opposition exists, for instance, in the reliability theory, where only two states (operable and failure) are possible.

In the mid XX century, it was still quite difficult to get used to the possibility of quantitative reliability assessment. However, by 70s, typical definition of the probability of failure as a numerical value of objective possibility of such accidental event, using the frequency of failures became familiar standard for engineers and technical community.

It should be noted that mutual transformations from a language of propositions to the language of events and back are made in a way that each event is associated with a proposition on its occurrence, and this proposition is associated with an event which turns out to be true.

Contradictory relations (“white – non-white”) are used not only in reliability, but also in other areas (safety, survivability, validity, etc.). Professor Ya. Golota, criticizing the LPA, considers that the real world is a world of contrary relations (white – black). The author [24] writes that the logic of contrary relations is logic of objective reality, and the logic of contradictory relations is logic of ideal world. Of course, the whole rainbow of colors (hypotheses) may be placed between white and black and it is closer to the reality, however I still support the ideal concept which is simpler and more honest.

As a conclusion of the history of LPA development, I would like to express my hope that not only N.V. Khovanov, but also other mathematicians will appreciate contributions of G. Boole, P.S. Poretskii, S.N. Bernstein, V.I. Glivenko, S.A. Yanovskaya, Yu.V. Merekin on applicability of the mathematical logic to the probability theory, and will reflect their opinion in publications.

In reply to the critics of Koyalovich on general impossibility of the algebra of logic to give fruitful extra logic applications [4, p. 417], below I would like to give a short list of references, comprising 150 publications in international periodicals on assessment of structurally complex systems [22] and 23 publications (see Appendix 1) on various SDP algorithms.

Conclusion

Boole’s rationalization by P.S. Poretskii in 1886 [3] strongly contributed to development of the logical probabilistic analysis by defining mathematical mutual exclusivity of propositions using their orthogonalization.

The orthogonalization algorithm was opened for the second time by Yu.V. Merekin [14] in 1963. Since then, there is a need in practical LPA application for solving reliability, survivability, and safety problems, first in the Naval Forces [19], then in Russia [20] and later worldwide [21].

For the third time the orthogonalization was opened in 1973 by Italian scientists L.Fratta, U.G.Montanari [15] using Karnaugh maps, Disjoint Products notion, and Sum of Disjoint Products (SDP) algorithms.

Due to mutual exclusivity of some propositions, A.S. Mozhaev and authors have developed [25,26] an algebra of disjoint events group (DEG).

International LPA authors develop SDP algorithms (Annex 1) and application software for solving the reliability, survivability, and safety problems.

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Annex 1

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